Ancient Indian Thought Contributing to The Field of Mathematics and Science

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"*Many of the advances in the Sciences that we consider today to have been made in Europe were in fact made in India, centuries ago*."

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Abstract

Ancient India was a land of free-flowing ideas and thoughts in all possible directions about all possible aspects of the inner as well as the outer world making the Indian civilization as one of the unique civilizations in the world with a vibrant and comprehensive tradition of spiritual as well as natural science. The sages and seers of this land who contributed to this rich culture as great thinkers were also scholars and scientists in their capacity. Almost all the prime aspects of human knowledge apart from spirituality like mathematics, astronomy, physics, chemistry, medicine and the practical procedures in which this knowledge was put into practice like metallurgy, architecture, shipbuilding and surgery etc. were covered in great detail by this science and technology in ancient India. Intrinsic fundamental concepts and principles of modern science have been provided with a foundation by these numerous postulates and scientific methods. While some of these ground breaking contributions have been acknowledged by the world body, some are still unknown to most.

Here in this article, we would discuss the contributions to the field of Mathematics and Physics.

Key Words: *Ancient Indian scientific thought, Origins of mathematics in India, Concept of zero and decimal system, Vedic and classical scientific knowledge, Contributions to astronomy and physics, Integration of philosophy and science.*

Mathematics

It is now generally accepted that India is the birthplace of several mathematical concepts including 'zero', numerical notations, decimal system, binary numbers; Fibonacci numbers, square root, cube root of numbers, algebra, algorithms, studies of infinite series, convergence, differentiation and iterative methods of solving nonlinear equations, ideas of calculus as well as geometry etc. Will Durant, an American historian (1885-1981) said that India is the mother of a lot of our mathematical concepts and philosophy. L. Basam, the Australian Indologist writes in his book "The miracle that India was..." that the world owes most to India in the field of mathematics to a level more advanced than that achieved by any other nation of antiquity. The success of Indian mathematics was primarily due to the fact that the Indian thought system was of a very high level in abstractions to think beyond the numerical quantity of objects and conceive clearly the abstract numbers. They could conceptualize the implication and significance of the most abstract entity in mathematics such as zero and infinity in its metaphysical forms as 'Sunya' and 'Ananta', which was very unique to Indian Culture. They invented the base ten number system with zero as a number, so as to be able to introduce numbers smaller than the smallest (called in Sanskrit as 'Anoraniyan') as well as numbers larger than the largest (called in Sanskrit as Mahato-mahiyan) which they needed to describe Nature with all its aspects starting with particles like atoms (Anus) to celestial bodies and the universe at large. The religious texts of the Vedic period provide evidence for the use of large numbers. Yajurveda Samhita (1200-900 BCE) mentions in its sacred mantra recitation at the end of food numbers invoking powers of ten from hundred (10^2) to an oblation rite (Anna-homa) as well as during asvamedha, trillion (10^{12}) and beyond. Thus, the roots of mathematics in ancient India can be traced back to the Vedic era as old as about 4000 years. Between 1000 BC to 1000 AD; a number of mathematical treatises had been written in India

One of the greatest and most important inventions of human mind is the concept of zero which owes its origin to the Indian Philosophy in connection with the idea of 'Sunya' which literally means void or nothingness that stands for the un-manifested unit source of all creations, the embodiment of infinite potentialities and the ground of being as depicted in the Vedic cosmology in the hymns of Nasadiya Sukta in Rig Veda. 'Zero' has emerged as a derivative symbol to represent this concept. The concept of 'Sunyata' or nothingness was also integral to Buddhist thinking according to Nagarjuna's Sunyavada. This was an idea which no western mathematician had ever thought of. Mathematician Aryabhatta of 5th Century AD, was the first person to use this present-day symbol (0) for zero as a number and a digit, whereas a small black circular patch () was used as a symbol of zero earlier. As early as 500 BC, Indians had also developed for each number from one to nine, a system of different symbols instead of alphabetic representations. By including the symbol of zero along with these nine symbols, Indians developed the ingenious method of writing a number, no matter how large or how small, only with these ten symbols. Aryabhatta in his Aryabhattiya has stated "Sthanat Sthanam Dasa Gunam Syat" which means from place to place each digit has a value ten times that of the preceeding one. In this system, called the 'decimal system', each digit while having its absolute value, receives its place value according to its position as well. Due to the simplicity of this decimal notation, it facilitated mathematical operations such as addition and subtraction etc. under the efforts of Aryabhatta. One of the earliest written evidence of the decimal place value system with the use of zero can be found in the Jaina cosmological text 'Lokavibhaga' written by the Jaina muni Saruanandin in 458AD (Saka era 380). In this text shunya (void) has been used to refer to zero. Laplace, the French mathematician and Philosopher therefore wrote - "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple now-a-days that its significance is no longer appreciated. Its simplicity lies in a way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of antiquity Archimedes and Apollonius."

This decimal system made arithmetic quite useful in practical inventions much faster and easier. Therefore, Albert Einstein also once remarked with his acknowledgement that - "We owe a lot to the ancient Indians, teaching us how to count, without which most modern scientific discoveries would have been impossible." This statement can be well appreciated if we just recollect the string of alpha-numeric Roman numbers having no zero and place value system to understand their limitations. Indian mathematicians invented negative numbers as well. Acharya Pingala; the Vedic scholar of 3rd or 2nd century BC was the author of the earliest known Sanskrit treatise on prosody (the study of poetic meters and verses) by the name 'Chandah Sastra'. This treatise presents the first known description of the binary numerical system in connection with the systematic enumeration of meters with fixed patterns of shorts (laghu) and long (guru) syllables. In modern discussions binary numbers are usually represented by using zero (0) and one (1). Pingala's notation was similar to Morse Code and he used the Sanskrit word 'Sunya' explicitly to refer to zero. This concept of binary numbers represented by '1' and '0' has now formed the corner stones of basic language for computer programs. Pingala is also credited with his work on Pascal's triangle (called meruprastara) as well as materials related to Fibonacci numbers called 'Matrameru'. Later on, the methods for the formation of these numbers in the sequence as (1, 1, 2, 3, 5, 8, 13 ......) and their implications were developed by mathematicians Virahanka, Gopala and Hemachandra, much before the Italian Mathematician Leonardo Fibonacci introduced this fascinating sequence to the western world in 13th century. Following the only method of the oral tradition of the time for the propagation of knowledge, be it spiritual or scientific, the sages, seers and scholars composed slokas in poetic styles in Sanskrit according to the rules prescribed in Sanskrit prosody described in 'Chanda Shastra' of Pingal. This methodology was based on natural rhythms and arrangement of tones such that the 'Slokas' so composed would be pleasing to the ears and easy to be remembered for a long time. Thus, mathematical concepts and formulas including various ideas were written down as meaningful syllables in verses and slokas. Indians therefore invented the 'Katapayadi' system, where even mathematical numbers could be transcribed as words or verses.

Indian number systems, as it is believed, probably arrived in the Arab world in 773 CE with the diplomatic mission sent by Hindu rulers of Sind to the court of Caliph Al-Mansur and subsequently through the Arabic traders. This gave rise to the famous arithmetical text written by Al-Khwarizmi in around 820 CE, which contains a detailed exposition of Indian Mathematics including usefulness of zero. Al-Khwarizmi was a Persian Mathematician who developed a technique of calculation that became known as 'algorism'. In fact, in 7th century CE, Brahmagupta developed the 'Chakravala Method' to solve indeterminate quadratic equations including Pell's equation. This method identified as a cyclic algorithm now was later generalized for a wider range of equations by Jayadeva and was further refined by Bhaskara-II in his mathematical treatise 'Bijaganita'. Algebraic theories as well as other mathematical concepts that were prevalent in ancient India were collected and further developed by the famous Indian Mathematician Aryabhatta in 5th century CE, who lived in Pataliputra, the present-day Patna in Bihar. His treatise on Mathematics is the 'Aryabhattiya'. Aryabhatta (466-550 CE) in his Aryabhattiya described important fundamental principles of mathematics in 332 slokas covering areas like algebra, arithmetic, trigonometry etc. He obtained the value of ' π' correct upto four decimal places. The Kerala Mathematician Nilakantha at subsequent times wrote sophisticated explanations of irrationality of' π' before the west had heard of the concept. The classical period between 400-1600 CE is often known as the golden age of Indian Mathematics. This period saw Mathematicians such as Aryabhatta-I, Varahamihira, Brahmagupta, Bhaskar-I, Bhaskara-II, Madhava of Sangamagrama and Nilakantha Somayaji and many others. The treatise by the Persian mathematician Al-Khwarizmi which contained all these developments with due credits to these Indian sources, was translated into Latin under the title 'Algorithm's de numero Indorum' meaning the system of Indian numerals. A Mathematican in Arabic is called Hindsa, which means from India. The technique of calculation developed by Al-as 'algorism' later became the germ for the modern computer algorithms. The 'Bakhshali manuscripts' of seventy birch bark leaves dating back to the early 7th centuries of the Christian era discovered in 1881 in the village Bakhshali near Peshawar (of modern-day Pakistan) reveals Indian achievements with knowledge of fractions, simultaneous equations, quadratic equations, geometric progression and even Khwarizmi, known originally calculations of profit and loss etc.

Our forefathers can also be credited for their knowledge in geometry, trigonometry and in some way calculus as well. 14th century Kerali Mathematician Madhava along with others of his Kerala School, studied infinite series, differentials and iterative methods for solving non-linear equations and examined methods and ideals relating to differential calculus. Jyesthadeva (1500-1576 AD) of the Kerala School wrote 'Yuktibhasa' in malayalum language comprising all these ideas. Jyesthadeva presented proofs of most mathematical theorems and infinite series discovered earlier by Madhava and other Kerala School Mathematicians. In fact, the landmark in Indian Mathematics was the development of the series expansions for trigonometric functions like sine, cosine and arc-tangent etc by the mathematicians of Kerala school in 15th century CE. These remarkable works developed two centuries before the invention of calculus in Europe by Isaac Newton and Leibnitz, provided what is now considered as the first examples of power series. However, they did not formulate a systematic theory of differentiation and integration.

It would be worthwhile to mention about the Sulba Sutras, composed between 800 BC to 500 BC in Vedic Sanskrit mainly for a single theological requirement with rules for construction of sacrificial fire-altars. There are three Sulba Sutras out of which the best known is Boudhayana Sulba sutra composed by Baudhayana during 8th century BCE, which contains examples of Pythagorean triples such as: (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37). This also contains a statement of the Pythagorean theorem for the sides of the square as "The rope which is stretched across the diagonal of a square produces an area double the size of the original square." It also has a similar statement for the sides of a rectangle as "The rope stretched along the length of the diagonal of a rectangle makes an area which the horizontal and the vertical sides make together". Baudhayana also gives an expression for the square root of two accurate up to five decimal places of the true value 1.41421356.... The other two Sulba Sutras are the Manava Sulba Sutra composed by Manava (750-650 BCE) and the other Apastamba Sulba Sutra, composed by Apastamba (600 BCE) with contents similar to Baudhayana Sulba Sutra. It has been found that the Babylonian cuneiform tablet 'Plimpta - 322' written around 1850 BCE, contains fifteen Pythagorean triples with quite large entries (13500, 12709, 18541). This indicates that there was also sophisticated understanding of the topic in Mesopotamia in 1850 BCE. Since these tablets predate the Sulba Sutras period by several centuries, taking into account this contextual appearance of some triples, it may be reasonable to expect that similar understanding would have been there in India. As the main objective of Sulba Sutras was to describe the construction of sacrificial fire-altars and the geometric principles involved in them, the subject of Pythagorean triples, even if it has been understood with its basic principles, many still not have featured in detail with the general proof in Sulba Sutras. This could have been due to the style of exposition demanded by the ancient oral tradition. Hence 'Sutras' adopted extreme brevity by expressing everything in a highly compressed form through multiple means. With the increasing complexity of mathematics and other branches of science like Astronomy, both writing and computation were required. Consequently, many mathematical works began to be written down in manuscripts to be copied from generation to generation.

India today has the largest body of hand-written reading materials comprising about several million manuscripts of prose commentaries and treatises. Then only derivations and proofs became favored. Thus Bhaskara-II (1114-1185 CE) in his Lilavati Bhasya, Bijaganita and Griha Ganitam that he wrote, had given a proof of the Pythagorean theorem. He had also conceived of differential calculus with concepts of derivatives, differential co-efficient. He had also stated Rolle's theorem, a special case of mean value theorem which is one of most important theorems of calculus and analysis. Bhaskara-II had also developed the concept of infinity.

Physics

From the Vedic times around 3000 BC to 1000 BC ancient Indian sages and scholars ventured to analyze and understand the physical structure of the world. They considered that the material world of living and non-living bodies in their gross structure are constituted holistically by five basic elements called 'Pancha Mahabhootas' such as Khiti (earth), Apa (water), Teja (fire/energy), Marut (air), Vyoma (ether/space). They were associated with the five human sense perceptions such as earth with smell, air with feeling, fire with vision, water with taste and ether or space with sound. These ancient Indian Philosophers believed that except for ether/space, all other elements were physically palpable and hence composed of minuscule particles of matter. The last miniscule particle of matter which could not be further subdivided was termed as 'paramanu', the synonym for the Greek word 'atom'. These Philosophers considered these atoms to be indestructible and hence eternal. However, in a later time the Buddhists believed atoms to be minute objects invisible to the naked eye which come into being and vanish in an instant like flares. The Vaisheshika school of Philosophers believed the atoms are mere points in space. As these concepts were based on logical analysis and abstract speculation but not on experimentation or personal observations, they are greatly abstract and enmeshed with philosophy as well. The school of Philosophy which contributed to the development of the ideas of 'atom' was the Vaisheshika School described earlier in chapter-4. Sage Kashyap known as Kanada Muni of 6th century BC, who composed the Vaisheshika Sutra, was the proponent of this idea. Another Indian

Philosopher, a contemporary of Gautam Buddha, Pakudha Kaccayana had also propounded ideas about the atomic constitution of the material world.

Adherents of the Vaisheshika School of Philosophy founded by Kanada considered the atoms to be minute objects invisible to the naked eye and atoms of the same substance combined with each other to produce dyanuka (diatomic molecule) and tryanuka (triatomic molecules). They also believed that atoms could be combined in various ways to produce chemical changes in the presence of other factors such as heat. As an example of such a phenomenon, Kanada cited the blackening of earthen pots and ripening of fruit etc. According to Kanada, each substance is supposed to consist of four kinds of atoms out of which two kinds possess mass and the other two without mass.

Apart from the atomic postulations, Kanada also had ideas regarding the motion and rest of objects suggesting probably the same laws of motion attributed to Newton in the seventeenth century CE; more than two thousand years after him. This is because one finds in the Vaisheshika sutras, the verses regarding motion of objects as follows: -

"Vegah Nimitta Visheshat Karmano Jayate;
Vegah Nimittapekshyat Karmano Jayate,
Niyatadika kriya Prabandha Hetu,
Vegah Samayoga vishesha birodhi."

This means action on objects generates motion. The external action being in a direction causes the motion in the same direction. An equal and opposite action can neutralize the motion.

In the fifth chapter of Vaisheshika Sutra, Kanada mentions various empirical observations on natural phenomena such as falling of objects to the ground, rising of fire and heat upwards, the growth of grass upwards, the nature of rainfall and thunderstorms, the flow of liquids, the movement towards a magnet and many other such cases and inquisitively searches why these things happen. Thus, it seems physics was central to Kanada's assertion that all that is knowable is based on motion and therefore he attempted to integrate his observation with his ideas on atoms, molecules and their interactions in some rudimentary level.

In fact, Rig Veda asserted that gravitation is the cause that is responsible for holding the universe together. This was some twenty-four centuries before the anecdotal apple fell on Newton's head. The notion of gravitation or gurutrakarshan is found in Siddhantas, the world's earliest texts on astronomy and mathematics. 'Siddhanta Siromani' is one such text written by Bhaskara-II (1114-1185 CE) in which one can find the mention of gurutvakarshana in its Goladdhyaya-Bhubanakosha chapter as:

"Marudhalo Bhurachala Swabhabato yato
Bichitrabata-bastu Saktyah.
Aakrustisaktischa mahitaya yat khastam,
Gurutwabhimukham Swasakttya.
Akrudayate Tatpattobabhati
Samasamntat kwa patatwiyam khe."

This means that each has the power of attraction by which it attracts material bodies towards it and so material bodies fall down on earth, when this power of attraction is uniform in all directions in the sky, then no object falls.

For this reason, the planetary systems and other stars and planets maintain their locations and motion in the sky. It has also been said that prior to Bhaskara-II, it was Varahamihira (505-587 CE), another Astronomer and mathematician of Siddhantic tradition who thought of the concept of gravity by claiming that there must be a force which might be keeping bodies stuck to earth and also keeping the heavenly bodies at specific places. Brahmagupta, another well-known mathematician of the 7th century had also commented on the concept of gravity as "Bodies fall towards the earth as it is like the earth to attract bodies, just as it is like water to flow." Therefore Dick Teresi, the American writer of the book 'Lost Discoveries', a comprehensive study of the ancient non-western foundation of modern sciences, spells out clearly as:

"Two hundred years before Phythagoras,
Philosophers in northern India had
understood that gravitation held
the solar system together, and that
therefore the Sun, the most massive object,
Had to be at its centre."

Aryabhatta, the one credited with the discovery of zero as the numeral, was also the first individual in 499 CE to explain that the daily rotation of the earth on its axis is the reason for the daily rising and setting of the Sun. Thus, he was the proponent of the helio-centric theory for the solar system. He conceived of the elliptical orbits of the planets thousand years before Kepler in the West assumed planetary orbits to be circular. Aryabhatta even came to the same conclusion. Before Kepler, Europeans estimated the value of the year as 365 days, six hours, 12 minutes and 30 seconds, only a few minutes off from the present correct value (365 days and six hours). This mathematical genius also made predictions of the solar and lunar eclipses as well as estimated the distance between the earth and the moon. The translation of Aryabhattiya into Latin in the thirteenth century taught Europeans a great deal revealing to them that Indians had known things that Europe would learn only a millennium after.

The Vedic civilization subscribed to the idea of a spherical earth at a time when everyone else, even the Greeks; assumed the earth to be flat. By the fifth century CE, Indians had calculated that the age of the earth was 4.3 billion years. But as late as the nineteenth century, English scientists believed the earth to be only a million years old. It is only in the late twentieth century that the western scientists have come to estimate it to be 4.6 billion years. This was in the aftermath of the first American Apollo mission to the moon that brought back the moon-rock to be analyzed to give this result which was highlighted in 1969 in American media comparing this outcome with ancient India's almost accurate estimate.

There has been ample archeological evidence for ancient India's use of 'practical mathematics' not only in measuring time on the basis of periodical cosmic events like earth's rotation or orbital motion etc. but also in standardized measurements for weights as well as length. Excavations at Harappa, Mohenjo-Daro and other sites of Indus Valley Civilization have uncovered bricks whose dimensions were in proportion 4:2:1; considered favorable for the stability of brick structures. People of Indus Valley civilization used the standardized system of weights based on the ratios: $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{2}$, 1, 2, 5, 10, 20, 50, 100, 200 and 500 with the unit weight equaling approximately 28 grams (approximately equal to one British Ounce). They mass produced weights in regular geometric shapes such as hexahedra, barrels, cones, and cylinders using their basic knowledge of geometry. They also used standardized measurement of length to a high degree of accuracy. They designed a ruler, the Mahenjodaro ruler, whose unit of length was approximately 1.32 inches or 3.4 cm which was divided into ten equal parts. The bricks manufactured at that time often had dimensions that were integral multiples of this unit of length. Hollow cylindrical objects made of shell and found at Lothal (2200 BCE) and Dholavira are demonstrated to also have the ability to also measure the angles in a plane as well as to measure the position of stars for navigation.

It is quite amazing to find that ancient Indians starting from the Vedic era had introduced various names for the units and sub-units of length or distance as well as time. As for example the Mokshya dharma parva of Shanti Parva in Mahabharat describes the units of time including Nimisha as follows. Accordingly, 1 diva Ratri (Day-Night) which is 24 hours as we know today was divided into 30 Muhurtas, I Muhurta was 30.3 kala; 1 kala was 30 kashta; 1 kasta was 15 Nimisha. 1 Nimisha is the time duration for the wink of an eye; which from the above relations can be worked out to be a recursive decimal in seconds as:

1 Nimisha = 0.2112, second.

Similarly, a unit measure of length or distance was taken as a 'Yojana', which has been defined in Vishnu Purana (Chapter 6 of Book 1) an ancient Vedic text in the following manner. If one starts with a standard subunit of length measure to be 1 Angula (1 finger length approximately 4 inch) then 6 Angula is 1 Pada, 2 Pada is 1 Vitasti, 2 Vitasti is 1 Hasta (cubit = 1½ feet), 4 Hastas is 1 Danda or Purusha (a man's height = 6 ft), 2000 Dandas is 1 Gavyutis (distance to which a cow's mowing can be heard = 12000 ft) and 4 Gavyutis is 1 Yojana which is approximately 9.09 miles. Working downwards from 1 Angula, the further sub-units are also defined in the following manners. 1 Angula which is 1.89 cm is 10 Yavas (barley grain of middle size), 1 Yava is 10 Yavodaras (heart of barely), 1 Yavodara is 10 Yukas, 1 Yuka is 10 Likhsha, 1 Likhsha is 10 Balagras (Hair's tip), 1 Balagra is 10 Mahirajas, 1 Mahiraja (Particle of dust) is 10 Trasarenu, 1 Trasarenu is 10 Parasukshma, 1 Parasukshma is 10 Paramanu. Thus, one can find out a rough estimate of the atomic dimension to be 1.89x10cm which is rather one order of magnitude smaller than what we know today in Physics to be of the order of Angstrom units (108cm) However there is another quite interesting estimate one can arrive at regarding the speed of light on the basis of a Rigvedic hymn (50th hymn in book 1 of Rig Veda), which is:

Taranir Vishvadarshato Jyotishkradasi Surya
Vishvama bhaasirochanam
Tatha cha Smaryate yojanam
Shahasre dve dve sate dve cha yojana
Ekena niminshardhena kramamana."

Which means;

"Swift and all beautiful art thou
O' Surya, maker of the light;
Illuminating all the radiant realm.
It is remembered here that this light
traverses 2202 Yojanas in half a nimisha."

Sayanacharya, who was a minister in the court of Buka of the great Vijaya nagar empire of Karnataka in South India in early 14th century commenting on this verse in his Rigvedic commentary has pointed out its significance in estimating the speed of light. If one takes the time unit Nimisha = 0.2112 second and the distance unit Yojana = 9.09 miles as found according to the above-mentioned ancient texts; then 2202 yojana in 1½ Nimisha of travelling would mean a speed of light 2202 x 9.09 miles per 0.1056 seconds. Which means the speed of light so calculated would be:

$c = \frac{2202\ x\ 9.09}{0.1056}$ = miles/second

= 189547 miles / second

As per the presently known value of the speed of light, c=186000 miles/second. This is amazingly so close to the accurate value that was revealed to our ancestors several thousand years before modern science could realize it through centuries of various attempts using different experimental techniques besides the theoretical calculation based on Maxwell's identification of light as an electromagnetic wave.

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